



Biological Dynamics and Semiotic Modeling of Multi-Species Trophic Interactions Using Interactive Visualization Tools: A GeoGebra-Mediated Intervention on Radical and Negative Integer Power Functions in Secondary Education

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Abstract

Several function families, including radical and those with negative integer exponents, play an important role in biological contexts where dynamics are modeled, ranging from allometric scaling to multi- species trophic interactions. These functions are important in various models of biological dynamics, from the study of allometric scaling to the study of multi- species trophic interactions, where they are observed simultaneously across several representations. However, the teaching of functions in secondary education remains largely on the algebraic register. This study investigates inter- register coordination and any changes that occur when an interactive visualization tool, GeoGebra, is used to study these two little- explored function families. A single- group pre- experimental pretest–posttest design was implemented with 16 first- grade secondary students at a private school in Ecuador, using a quantitative approach. In four workshops of five classes each, four registers (verbal, algebraic, tabular, graphical) were explained, and conversion tasks were set between the registers, while intra- register treatment tasks were facilitated with the aid of GeoGebra. Instruments included a pre- and post- intervention 24- item questionnaire, an observation sheet arranged by semiotic operation, and a rubric developed from 5 criteria, all of which were expert- judged. None of the respondents met the lower limit at the pretest. The Wilcoxon signed- rank test revealed a statistically significant difference ($Z = 3.516$, $p < .001$) with a large effect size ($r = .62$), and all 16 students improved after the intervention. The unevenness of the relationship between the registers was examined as follows: The algebraic- to- tabular was the easiest to do, while the intra- register was the most difficult. The results indicate that interactive visualization is useful for making the conversions necessary to read a model of a biological phenomenon, whereas symbolic treatment, including explicit instruction, cannot. These results should be interpreted as evidence of association rather than causation because there was no control group.

Keywords

Biological dynamics; semiotic modeling; trophic interactions; interactive visualization tools; GeoGebra; radical functions; negative integer power function; secondary Education.

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Introduction

Dealing with nonlinear relationships in living systems and many of the quantitative regularities of ecology take the form of power and rational functions. The metabolic theory of ecology (Brown et al., 2004), with its predictions for population growth, energy flow, and trophic structure, is based on this radical relationship between metabolic rate and body mass raised to a fractional power of about three-quarters. The predator-prey functional relationship is characterized by a rational form with a negative integer power, which results in asymptotic behavior as prey density increases (Holling, 1959). The ability to model multi-species trophic interactions therefore benefits from navigating simultaneously across several sets of models, as provided by interactive visualization tools. The essential properties of functions (domain restrictions, asymptotes, discontinuities) are best understood by comparing several models at once.

In secondary school, instruction is still stuck in the algebraic register; students can add, subtract, multiply, and divide using graphics and solve equations, but they cannot see the general form of a function or understand why some values are not in a domain. This is explained by the theory of semiotic registers of representation. In his argument, Duval (2006) declared that mathematical objects can only be attained through semiotic representations and that understanding can only be achieved by a transition between them. The study explained how the conversion of a representation in one register differs from the treatment of that representation, which is the transfer to another register while preserving its reference, and demonstrated that the latter focused the difficulties and that the cognitive cost of the conversion increased with a loss in congruence between the registers. Zahner et al. (2017) found a systematic variation in student performance depending on the pair of representations that were used, and Erbilgin and Gningue (2023) demonstrated that the availability of the representation influenced the unfolding of mathematical practices. Semiotic modeling of a biological system here is a sequence of transformations: first from a discipline of "how interaction is talked about"—namely verbal description—then to a mathematical model of it—namely algebraic modeling—and then to tables of values of the simulated actions—namely tables—and then to resulting graphs—namely diagrams.

The key advantage of using GeoGebra is that changing one representation (Table, Graph, or Algebraic) automatically updates the other two. However, realizing this benefit is not automatic. Ainsworth (2006) demonstrated that multiple representations serve different purposes, and van der Meij and de Jong (2006) found that learning gains depend on students' internalization of translations between representations, not on external cues from the environment. Despite this, less prescriptive tasks led students to exploit GeoGebra more widely (Olsson, 2019), and Andrés, Coronel, Di Rico, Luna, and Sessa (2021) reported that coordinating on-screen representations was not spontaneous and must be carefully facilitated by the teacher. While controlled studies have found positive outcomes for GeoGebra in learning functions (Božić et al., 2021; Takači et al., 2015; Zulnaidi et al., 2020), these studies do not specify which semiotic operations improve, nor do more extensive reviews of the literature (Bray & Tangney, 2017; Yohannes & Chen, 2023). This procedural boundary is further exacerbated by a content gap. While linear, quadratic, and trigonometric functions, and calculus have received attention, the building blocks of allometric and trophic models, namely radical and negative integer functions, have received little attention. These functions have low congruence across registers; the domain restriction of a radical function is an algebraic inequality but is represented graphically by an empty area on the plane, and the asymptote of a function with a negative integer power is conveniently masked by a tabular display of the power function's values, which indicate continuity but, in reality, do not.

In Ecuador, two different families are introduced during the first year of upper secondary school, but an in-depth study of teaching them using digital tools remains incipient nationally. In this study, therefore, the following intervention model is constructed and tested using GeoGebra tools as a semiotic basis to model biological dynamics, which consists of four groups of registers from which treatment and conversion can be organized. The research question is: What is the relationship between the learning of first year upper secondary power functions with negative integer exponents and such an intervention? There are three operational questions: What is the students' starting point for their registers? How do they execute their processes, that is, what are their routines and stances in relation to their semiotic operations during the implementation? What differences in performance occur before and after the intervention?

Materials and Methods

The study is conducted within a positivist paradigm using a quantitative approach, with the dependent variable measured and hypotheses tested deductively. It is designed as a pre-experimental test with one group tested pre-

and post-intervention. This modality was determined by the actual conditions of the participating institution, where a group taking the same content at the same time but not receiving the intervention was not available. This design is intended to clarify what it allows and what it does not allow. The absence of a control group or random assignment means other possible explanations for the change in behavior (maturation, parallel instruction, events that occurred during the period) cannot be ruled out. Therefore, the scope is specified as descriptive and correlational, rather than explanatory, indicating that the results are considered an association between the time of measurement and the results, not a cause-and-effect relationship.

The sample comprised 69 students from Ecuador, in an upper secondary school of a private institution. The sample consisted of 16 first-year students who were drawn by nonprobability sampling for convenience, the schedule of the required courses, and authorization by the institution. This was a whole group sample and the results are limited to populations that have similar characteristics. The Four Components of the Intervention Model. The first sets the registers at play (verbal, algebraic, tabular, and graphical) and their 12 possible ordered conversion pairs, four of which were given special consideration for curricular and congruence arguments. The second follows a line of reasoning based on anticipation: the student guesses what will appear in the target register before it is displayed in GeoGebra, acknowledging that automatic linking could negate the effort put into the translation (van der Meij & de Jong, 2006). The third one slowly decreases the guidance of the sequence as it goes along, following the way as suggested by Olsson (2019). The fourth uses whole-class discussion about the object's invariance across its representations, as suggested by Andrés et al. (2021). Table 1 shows the correspondences among the goals of the study, the prioritized semiotic operations, and the instruments employed to document the implemented operations.

Table 1. Correspondence Among Objectives, Prioritized Semiotic Operations, and Data Collection Instruments

Objective	Prioritized semiotic operation	Instrument	Timing
Diagnose initial status	Recognition of registers; algebraic treatment	Questionnaire (24 items)	Pre
Implement the model with radical functions	Algebraic $\rightarrow$ tabular conversion; tabular $\rightarrow$ graphical conversion	Observation sheet; rubric	During
Implement the model with negative integer power functions	Verbal $\rightarrow$ algebraic conversion; tabular $\rightarrow$ graphical conversion	Observation sheet; rubric	During
Compare pre- and post-intervention performance	Full set of treatments and conversions	Questionnaire (24 items)	Post

Note. Prepared by the authors based on the intervention design. Conversion pairs are expressed as source register $\rightarrow$ target register, following Duval's (2006) nomenclature.

A total of three instruments are utilized, linked with a specific objective. The pretest–posttest questionnaire had 24 items pertaining to functions involving radical and negative integer exponents, with a sample item reading: "On the graph of a function, $f(x) = x5/3$, which has a negative slope, the point (9, y) is located above the x-axis. A sample item of the pretest–posttest questionnaire was "On the graph of a function, $f(x) = x5/3$, where the slope of $f(x)$ is negative, the point (9, y) is above the x-axis". It was created from the learnt curriculum for this level and it took 3 class periods for it to be administered. Bilateral pre- and post-administration of the same instrument allows for direct comparability of the scores at the result; however, it gives the risk of practice effects, which is discussed further. Throughout the workshops, the use of parameters related to interaction with the tool, application of concepts, use of registers, collaboration, and positive attitudes towards the task were qualitatively evaluated with three answers (yes, no, partly) to be recorded on the observation sheet. The rubric evaluated the following five performance areas: mathematical concepts, Problem-solving process, Use of GeoGebra, Clarity in the presentation of work, and Participation and collaboration. The grade levels were assigned based on the official grade level system as specified in the Table 2.

Table 2. Coding of the Student Performance Rating Scale

Code	Qualitative scale	Quantitative scale
DAR	Masters the required learning	9.00–10.00
AAR	Achieves the required learning	7.00–8.99
PAAR	Is close to achieving the required learning	4.01–6.99
NAAR	Does not achieve the required learning	≤ 4.00

Note. It is adapted from the scale of student assessment used by the Ministerio de Educación del Ecuador (2016). The original Spanish (ACVAR) name of the official scale is preserved. To pass you need 7.00 points.

Content validity was determined using experts' opinions. The instruments were framed and rated by three experts, including two with a specialization in mathematics education and a member of the mathematics department of the involved institution, using a matrix of 3 indicators: pertinence, relevance, and clarity. The score was unanimously good on all three (Table 3).

Table 3. Content Validity of the Instruments by Expert Judgment

Expert	Pertinence	Relevance	Clarity	Verdict
Expert 1	Yes	Yes	Yes	Applicable
Expert 2	Yes	Yes	Yes	Applicable
Expert 3	Yes	Yes	Yes	Applicable

Note. The questionnaire, observation sheet, and rubric were rated by all three experts. The verdict of 'Applicable' allows the instrument to be used in the same format.

Cronbach's alpha coefficient was calculated for the 24 items of the instrument and was found to be .716, indicating a high degree of reliability for the questionnaire. This suggests moderate internal consistency, adequate for the exploratory use of the instrument at the class level but inadequate for individual decision-making. Refinement of the least item-total correlation items is an outstanding work item for future use of the instrument. Intervention was provided in 2 five-class periods per workshop, with the first workshop on radical functions and the second on negative integer power functions. Consequently, the conceptual explanations of each workshop were followed by guided exploration activities in GeoGebra, similar to the anticipation principle described above; that is, students first wrote down their predictions about the target register and then tried to confirm them using the software. The first workshop allowed a comparison between the algebraic definition of the domain using inequalities and the graphical representation as a "hole" in the plane when working with square roots of linear expressions. The slider function of GeoGebra was used to explore the effects of changing the parameters on the curve by reading both the changes in the symbolic expression and the changes in the curve's graph. The second workshop involved the study of a class of functions that present a problem of the asymptote: students were asked to prepare tables of values to estimate the behavior of the function in the immediate vicinity of the point of discontinuity, make a conjecture about the behavior of the function at the discontinuity, and then use the program to obtain a representation of the function and compare it with their conjecture.

There was logic to this sequence, since a table of discrete values can convey a sense of continuity, and the juxtaposition with the graph creates a cognitive conflict that is of educational interest. Two contextual constraints were considered: the loss of some sessions due to institutional activities planned within the school routines and the variability in students' prior exposure to devices, which warranted allowing time for a first meeting with the interface during the first workshop. Data analysis was carried out using standard statistical software. Three substantive issues were taken into account when choosing the test. First, the design includes repeated measures on the same group, so there is dependence between data observed before and after the design. Second, the dependent variable is measured on an ordinal scale with four levels—procedures that assume interval-level measurement dispute this. Third, there are only 16 cases, so there is insufficient power to test for normality of differences, and it would be reasonable to apply the central limit theorem.

The determining principle is not so much the size of the sample per se but, more, the level of measurement and the inability to maintain the distributional assumption. Therefore, the Wilcoxon signed-rank test for related samples was used, which tests the median of the differences between corresponding samples and does not require the assumption of normality; both the asymptotic approximation and the exact significance were considered due to the small number of samples. The final step involved analyzing the Es according to Cohen's (1988) conventional size

or scale designations: .10 (small), .30 (medium), and .50 or greater (large). As a secondary measure of consistency of change in direction, a series of matched- pairs rank- biserial correlations was also calculated. The null hypothesis was that the median of posttest - pretest differences is 0, while the research hypothesis was that it is not equal to 0, and a two- tailed significance level of .05 was used. Ethical precautions were addressed at the point of entry in the methodology, given the doubly asymmetrical process between adults and another group of students, who are minors, and between adults and school, which is responsible for grading them. The framework was based on principles consolidated through state- of- the- art contemporary bioethics. Manzini (2000), who discusses the Declaration of Helsinki, insists that participants' welfare always comes first and that their dignity, privacy, and confidentiality of information are at stake. Brítez Silvero (2016) notes that the Belmont Report provided the basis for all research involving humans, and it is for this reason that there is a place for independent review, since no one, neither the researcher nor the human subject, is purely objective enough to evaluate the ethical integrity of a project. There is a documented history of informed consent being neglected, muffled, or withdrawn, stressing the importance of informed consent as a substantive safeguard, rather than an administrative occurrence [as Castro- Molina (2020) points out].

An application was submitted to the school authority on this basis, and informed consent was sought from students' legal representatives and assent from students—who were told in accessible language what the study was about, what activities would be conducted, and that participation was voluntary. Because the researcher was also the students' teacher, it was emphasized that they could drop out of the course at any time without affecting their grades, which was crucial, as one of the main ethical concerns in classroom studies. Data are anonymous (coded numerically) and reported only in aggregate. No student lost access to curricular content due to design considerations – the absence of a control group – and the ethical equivalent was that everyone received the intervention. Unfavorable as well as positive results are communicated with equal visibility due to the instrument's moderate reliability or continued problems with a particular operation.

Results of the Wilcoxon Signed-Rank Test for Related Samples

Statistic	Value
Total N	16
Negative ranks (post-test < pretest)	0
Positive ranks (post-test > pretest)	16
Ties	0
Sum of positive ranks (W^+)	136.000
Standard error	19.339
Standardized test statistic (Z)	3.516
Asymptotic significance (two-tailed)	$p < .001$
Effect size ($r = Z/\sqrt{N}$; $N = 32$)	.62
Rank-biserial correlation	1.00

Note. The value $W^+ = 136.000$ equals the theoretical maximum for $n = 16$, since $n(n + 1)/2 = 136$. Interpretation of Effect size: Cohen (1988): .10 small, .30 medium, and .50 or above was large.

The values highlight a detail that is usually overlooked in the report. The sum of positive ranks was exactly 136, the maximum possible sum for 16 pairs ($1+2+3...+16$). The statistic reflects what it should achieve, which explains that none of the 16 students declined or tied – each improved! It's this condition—not just the p-value—that makes the result robust rather than merely significant: if the p-value was significant, it might have been because x showed greater gains in a handful of cases and greater losses in a handful of other cases. The mean score (M) = 3.516 SDs, with two-tailed significance $< p.001$, standardized statistic.

For the repeated-measures design, the effect size was $r = .62$ —so far in the large range. Since there were no negative differences, the rank-biserial correlation reached unity, an arithmetically necessary result but not an independent estimate of magnitude, and is reported as an indication of the direction of consistency. The null hypothesis is therefore rejected, since the difference between the scores before and after the intervention, in this case the difference in performance for the group on the treatment and conversion tasks with radical and negative-integer power functions, was sufficiently large, and the intervened group performed significantly better than the non-

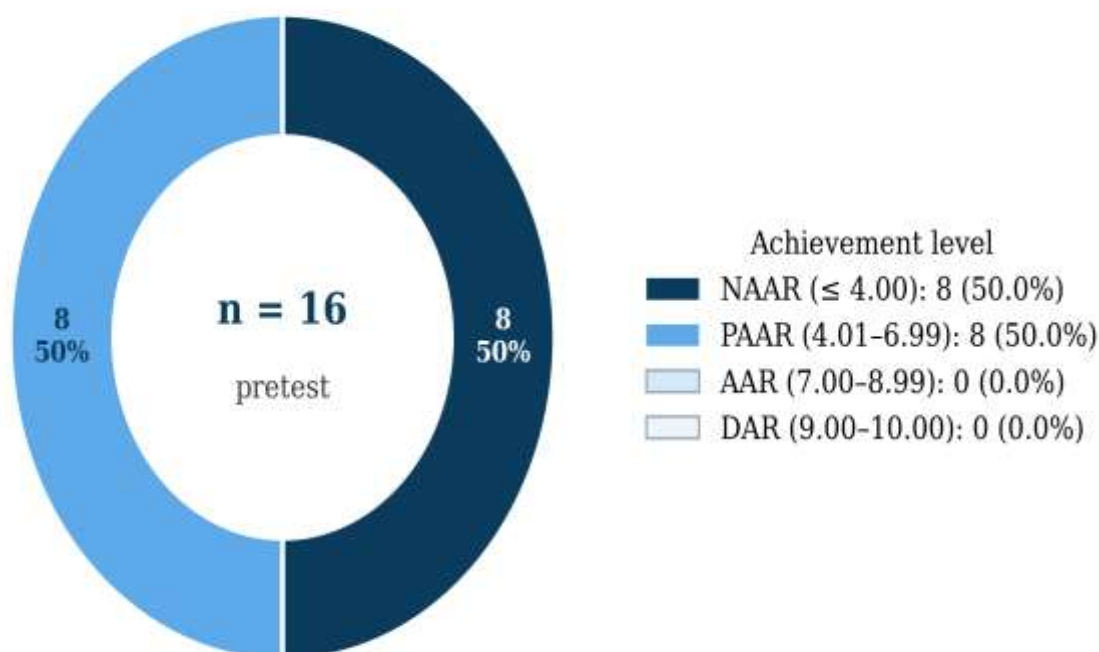
intervened group. Again, it is important to note that the test compares differences in medians (intra-individual differences), not differences in means, as is often overlooked in the applied literature. In force terms, the discovery relates to the accumulation of evidence. Studies by Božić et al. (2021) and Zulnadi et al. (2020) show positive results in function content for groups using GeoGebra, as implementation of collaborative work provides consistent results in calculus, according to Takači et al. (2015).

This convergence is comforting but should be tempered: In those studies, there were control groups to temper the results further than in the current one. This is not a statement of equality, rather it is evidence that is provided within different contexts, in a context that is away from the ones those studies have considered, is the contribution; it is not the power of design itself that is extended. A sense of caution is needed for causal interpretability. All participants improved, and the effect size is large, leaving room for strong statements, but not for the current study. Similar measures across both administrations, maturation over time, and the regularity of instruction in parallel are potential rival explanations of practice effects. However, there is a partial counter to the objection of practice effect: If improvements were primarily due to familiarity with the items, then an essentially consistent improvement effort would be expected, but what was observed was an improvement effort that varied by type of treatment, with smaller improvement attempts in intra-register treatments. This degree of heterogeneity is hard to account for by repetition, and is more consistent with a specific effect of the intervention. This is a suggestive argument, not a conclusive one.

Results and Discussion

All 16 students who were tested were PAAR or NAAR, with eight in each of those categories, and none met the PASS criterion of seven points (Figure 1).

Figure 1. Distribution of Achievement Levels at Pretest

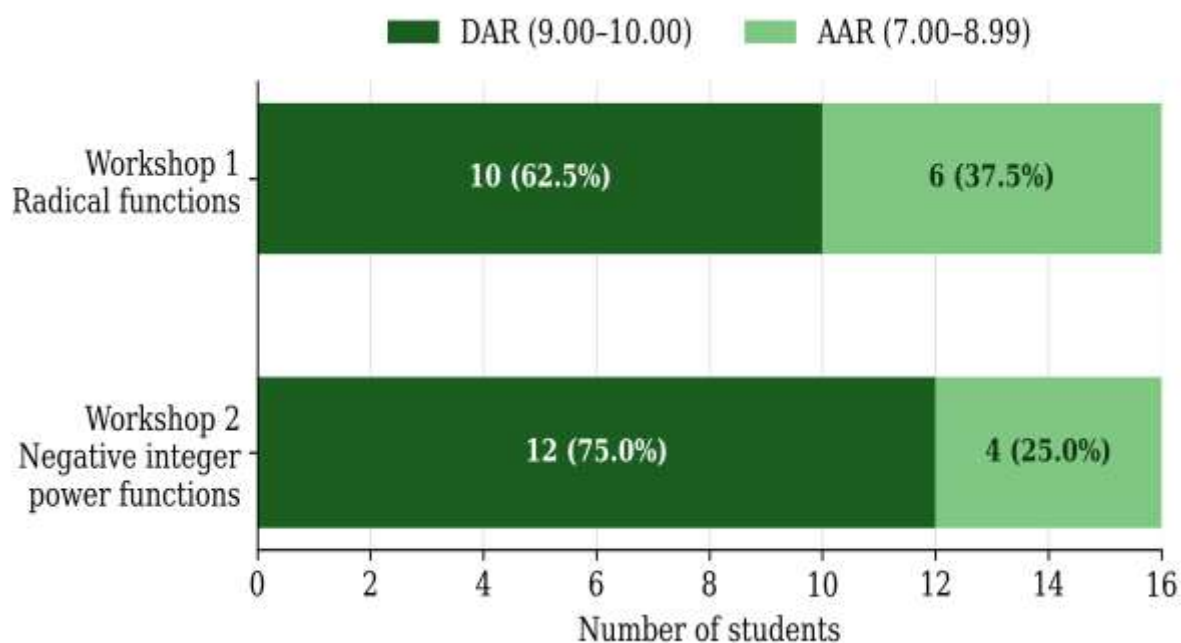


Note. $n = 16$. The values within the segments indicate the frequency (and the percentage) of the group. For AR and DR there were no cases. The grading criteria has been taken from the Ministerio de Educación del Ecuador (2016).

This number can be viewed in two ways. The more noticeable quantitative result: none of the students passed. The more interesting is the qualitative view, which comes from the item analysis. Errors were categorized by items that were similar in marking but different in difficulty. Correct answers tended to be consolidated in algebra problems involving elementary manipulation, and the errors tended to occur in problems involving moving towards the graph or understanding the meaning of a domain restriction. But the group was not completely lacking in tools: they had one; they didn't have the skill to coordinate it with the others. This mirrors the theory's calculation in cases where

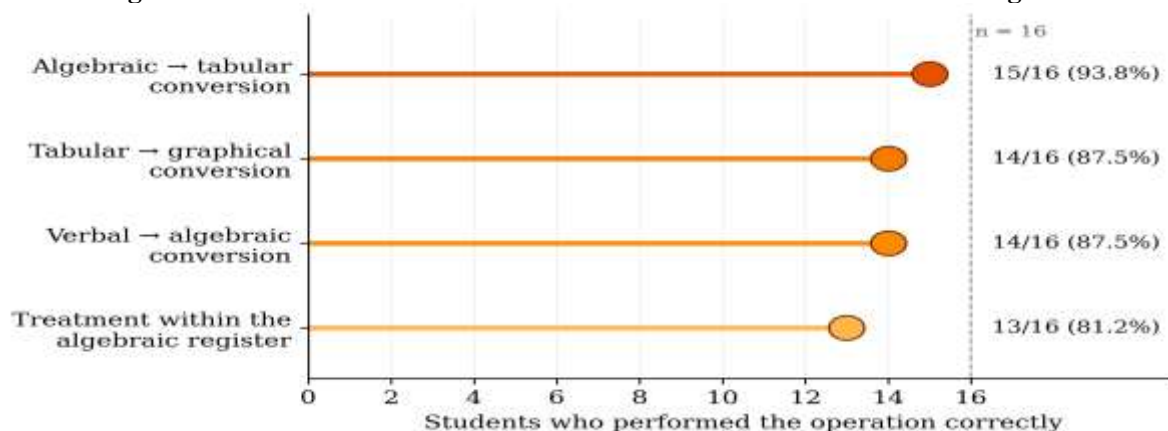
teaching has occurred and favors one register. This is supported by the fact that no students were reported at the AAR and DAR levels, which rules out interpreting individual differences in aptitude and points and suggests the issue concerned the students' instructional trajectory rather than a few lagging students. When considered in conjunction with the results of Yohannes and Chen (2023), who reported that their research focused mainly on geometry and calculus, this trend further illustrates that the symbolic register has been privileged in school teaching. The conclusion does not focus on students' deficits, but on a characteristic of the teaching tradition, which shifts intervention from the individual to the curricular level if the diagnosis is representative of similar cases. Rubric ratings for the two workshops demonstrate positive results and, more informatively, a progression from Workshop 1 to Workshop 2. Ten students in the workshop on radical functions and 6 in the workshop on negative integer power functions participated in the workshops at DAR, and 6 in the workshop on radical functions and 4 in the workshop on negative integer power functions reached the AAR level (Figure 2).

Figure 2. Performance Levels Attained on the Rubric in the Two Workshops



Note. $n = 16$ in each workshop. The rubric was based on five standards including: understanding of the mathematical concepts, problem-solving process, use of GeoGebra, clarity of presentation, and participation. None of the Grade 12 students were rated at PAAR/NAAR level.

The process by which two students advanced in the workshop from AAR to DAR introduces more than one interpretation; it should not be unnecessarily complicated. It is possible that the content of the second workshop was somehow more easily understood; it is hard to imagine this because discontinuity and asymptotes have more conceptual difficulties than domain restriction. What seems more likely is that, in the first workshop, some students didn't understand how to use the program interface for setting up a function in various registers, whereas this was almost unheard of in the second. This reading deals with Olsson (2019) in a manner that needs to be made particularly clear. The present finding corroborates Olsson's finding of a preference for reduced guidance for use of the software and quality of reasoning, but only after instrumental entry cost has been overcome. Autonomy that happens too early for the novice can be distracting or before their time and lead to the use of mental effort that is not being allocated to the mathematical content. The instructional implication is for the teacher to provide less and less guidance over time, based on mastery of the instrument, and to make the instrument's implementation cost part of the design, not noise. The most theory-relevant results are gained from the observation sheet, where there is a distinction between the various semiotic operations in performance. The number of students who executed the operations successfully in the workshops is given in Figure 3, and unsuccessful cases are given in Table 4, together with the cases that they did successfully.

Figure 3. Correct Execution of Treatments and Conversions Between Registers

Note. $n = 16$. Percentages are calculated over the total number of students observed. The source register $\rightarrow$ target register notation follows Duval's (2006) convention.

Table 4. Performance by Semiotic Operation During the Intervention

Semiotic operation	Performed correctly	Not achieved	Achievement rate
Algebraic $\rightarrow$ tabular conversion	15	1	93.8%
Tabular $\rightarrow$ graphical conversion	14	2	87.5%
Verbal $\rightarrow$ algebraic conversion	14	2	87.5%
Treatment within the algebraic register	13	3	81.2%

Note. $n = 16$. The data are from the two workshops recorded on the observation sheet. Operations are run in order: Highest to lowest achievement rate. There are three observations to be made in light of these data.

The algebraic–tabular conversion causes an unambiguous degradation in performance. This aligns with the idea of Congruence, which translates an expression into a table by repeatedly using information – but does not involve rearranging information. Say what you will, it is indeed the next step toward treatment, and students with algebraic strength do not doubt find performing it easy. Second, the verbal–algebraic conversion, theoretically the costliest as it has the least congruence, also caught up with the tabular–graphical conversion. This was unexpected. This may be because the verbal tasks were designed, to some extent, to minimize the need for reorganization, or because of the scaffolding effect of the verbal whole-class discussion in the model. Available information does not distinguish between them, and the point is listed as a line of inquiry. The third observation is the most incongruous: subjects treated at the algebraic register (not converted into any of the other registers) achieved the lowest score, lower than all conversions. This was indeed surprising, because it had been determined in the initial diagnosis that the algebraic register was the sure foundation for the group. The interpretations that were most coherent with the theory were that the tasks proposed in the workshops required a level of algebraic competence higher than the one that the group had, whereas the conversions were possible thanks to the group's dynamic (even in the task of handling negative exponents). To be brief, GeoGebra helped ease the conversion, but it couldn't take the place of symbolic skills, and where it fails, weakness returns. This explanation adds to van der Meij and de Jong's (2006) argument on the constraints of external representational support and refines the perspective in an area they do not cover. They predicted that automatic tying of representations could save students from translating between representations, whereas the present data indicate a counterintuitive and probably more disturbing finding: that this saving can obscure ongoing weaknesses in nonrepresentational operations not represented in the software. Just because a student obtains a correct conversion in the program does not mean he or she has gained conversion competence, and therefore subsequent assessment without the program should be taken as such. To us, this is the principal practical takeaway from the study.

The most theoretically significant finding of the study is this operation-level profile, which represents the algebraic–tabular conversion at the top and the intra-register treatment at the bottom, and is congruent with previous work by Zahner et al. (2017), who found that student performance was systematically different based on the pair of representations studied in the cluster analysis. The two studies agree that representational competence is not one

unitary skill but a collection of moderately independent skills; and, crucially, for assessment design, a global score hides right in front of them exactly what they need to intervene. But Andrés et al. (2021) also offer a complementary warning: Coordinating the representations that appear on a screen at the same time does not happen naturally. However, given that caution in this regard, the model proposed here involved whole class discussion about the invariance of the object: favoring performance on the conversions indicated that this part of the model played its intended function, but the function of this element was not separated from that of the other elements in the design. This profile is also directly connected to the semiotic modeling of the dynamics in the biological field.

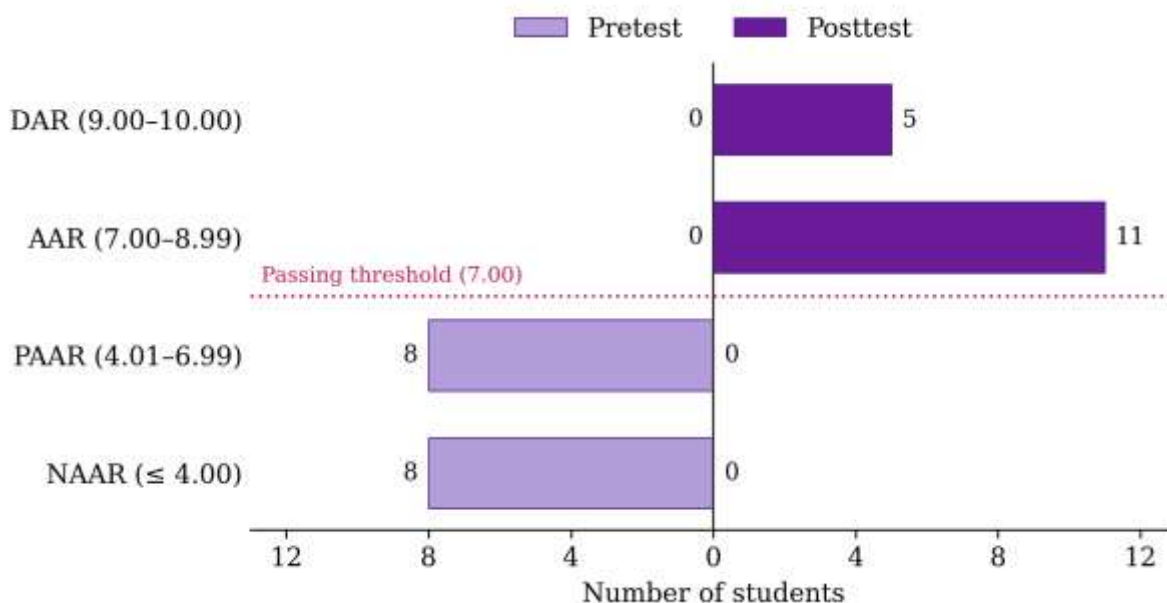
The fact that the tabular-graphical conversion (a table of discrete values must be interpreted as representing an incomplete graph of a curve with an asymptote) shows high achievement in this operation indicates that interactive visualization could be helpful for exactly the content that population and trophic models require for reading. However, its relatively ineffective intra-register treatment suggests that deriving and comparing models using algebra—manipulating allometric expressions with fractional and negative exponents—can't be achieved exclusively by the visualization tools. Instructing an exemplifying sequence of interactions among multiple species would then require a sequence of dynamic exploration and direct symbolic transformation. The effects of the posttest were total, reversing the pretest distribution. There were 11 students at the AAR level and five at the DAR level, and none below the students' passing level. To compare the two measurement times, see Table 5 and Figure 4.

Table 5. Distribution of Achievement Levels at Pretest and Posttest

Achievement level	Pretest (f)	Pretest (%)	Posttest (f)	Posttest (%)
DAR (9.00–10.00)	0	0.0%	5	31.25%
AAR (7.00–8.99)	0	0.0%	11	68.75%
PAAR (4.01–6.99)	8	50.0%	0	0.0%
NAAR (≤ 4.00)	8	50.0%	0	0.0%
Total	16	100.0%	16	100.0%

Note. $n = 16$. Median changes from NAAR to AAR at post-test while it is between NAAR and PAAR at pre-test. f = absolute frequency.

Figure 4. Comparison of the Distribution of Achievement Levels Before and After the Intervention



Note. $n = 16$. Bars indicate absolute frequencies, with pretests on the left of the axis and post-tests on the right. The passing criterion is given by a dotted line. No overlap between the distributions anywhere; complete group shift.

Before addressing the inferential test, it was important to emphasize one aspect of this movement: both distributions have nothing in common in terms of levels. All scores for the group were in the failing range, and none crossed into the passing range. This was of the order of complete separation, which is rarely reached if it is anticipated, especially

if it is anticipated by the statistical test (as in this example; Table 6).

Limitations and Scope for Future Research

The most important drawback is the lack of a control group, which makes it difficult to rule out other closely related explanations of the observed change and limits conclusions to association. Another downside to using the same instrument is the "practice effect" that only parallel forms could counteract. Third, the researcher was also the teacher who applied the intervention and completed the copying and observation instruments, which may be a source of bias; external observers should be included or scoring should be blind to minimize this potential bias in future replications. Fourth, because the sample size and convenience sampling from one institution restricted the statistical power and generalizability of the results, this study should be repeated in other institutions. The overall moderate reliability of the questionnaire also indicates the need to revise some items with low item–total correlations, and the lack of a delayed measurement both improves the likelihood that observed performance will be maintained over time and strengthens the overall internal consistency and reliability of this measure. Fifth, the present study lacks an evaluation of performance without the tool; it cannot be determined whether a new competence has been acquired or whether the software was used to execute a skill that was already developed without the aid of the tool. The distinction discussed in the present study appears to be crucial in this context.

Finally, the workshop problems were formulated in mathematical terms only; while the application of the model is given in the biological context, it was presented as a biological problem, no biological data or any trophic model was used, and the transfer to biological modeling has not been proven. Such restrictions suggest a number of lines of future research. First, create a second model that includes a quasi-experimental design and a control group with parallel forms of the instrument, as well as a double measure of delayed retention. The second is to assess the process without the tool and with the tool to estimate the percentage that is not dependent on the tool. The third, central, and important for framing this article, is to set the biological context to which the intervention is applied; for instance, students can create predator–prey and multi-species models using sliders from the GeoGebra Applet, where they control the amount eaten, handling time, and variation involved in the consumption, and to check if register coordination is transferred to the understanding of the trophic dynamics. Next, more generally, implement disaggregation by semiotic operation as a standard reporting method for research on how functions are learned using technology, because current evidence indicates that aggregating measures hides variation that has didactic significance. Additionally, the factorial designs would allow for disentangling the effects of each element of the intervention model, especially the whole-class discussion and anticipation principle.

Conclusion

The initial diagnosis revealed a consistent group in which all participants performed below the passing level; correct responses were grouped in the algebraic register, while faulty responses were associated with tasks that required coordinating responses in the algebraic register with the other registers. This configuration corroborates general findings from register theory about teaching focused on a single representational system, particularly for radical functions and functions with negative integer exponents. Adoption of the intervention model produced positive performance across the four prioritized operations of the semiotic system, with an uneven profile. The algebraic–tabular conversion was easiest, and the algebraic register was the most resistant, a finding that is the study's most particular contribution. This profile highlights that representing a dynamic environment is well suited to movement between registers, but it does not compensate for a lack of representation of symbolic operations that cannot be sustained in a dynamic environment. This is a warning that has a direct bearing on how we design instruction—that is, on how we prepare people to model the dynamics of biological systems, where the reading of asymptotic graphs and the symbolic manipulation of power relationships are essential. The comparison of the two measurements showed a statistically significant, large effect size improvement in every participant, with no cases of 'declining'. The strength of the pattern, combined with evidence of heterogeneity across operation types, is consistent with an intervention effect (but not causation, given the nature of the design).

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